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DEVELOPING NETWORK LOCATION MODEL IN UNCERTAINTY MODE (ROBUST MODE)

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ABSTRACT: In this research, facility location problem-network design under uncertainty robust mode has been discussed. In this regard a model will be developed, so that the uncertainty in parameters such as demand and problem's various costs considered. Facility location-network design, unlike classical facility location models, which are assumed that network structure is pre-defined and pre-specified, will also decide on the structure of the network. This has been in many actual applications such as road network, communication systems and, etc. and finding facility location and main network designing simultaneously has deemed important and the need for simultaneous design and optimization models to meet the mentioned items is felt. Different approaches have been developed in the uncertainty optimization literature. Amongst them, robust and stochastic optimizations are well known. To deal with uncertainty and problem modeling, in this research robust optimization approach have been used. In addition, by using generated random samples, the proposed model has been tested and computational analysis is presented for various parameters.

Keywords: Facility Location. Network Design. Robust Optimization (Solid). Minimizing the Maximum Regret.

1 INTRODUCTION

Facility location problems in found an important place in operational research literature from the 1960s. In general, the term location refers to modeling, formulating and solving problems that we can define them the best of the best by placing the facility in available space.

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These problems explore how we can locate a set of facilities, physically, as an objective function is optimized under a set of restrictions.

Recently done studies indicated that network location models are widely have been used to layout facilities in the private and public sectors, up to now. In most models proposed for facility location, network structure is pre-specified and links between nodes are pre-defined. While in many real-world problems, this issue may not be acceptable. Therefore, this issue is important. So, finding facility location and designing main network simultaneously, in many real-world problems may be important and the need for simultaneous design and optimization models to meet the mentioned items is felt.

Location models were first presented in 1996 in certainty mode. In this paper, the problem of facility location with considering network design in uncertainty mode will be developed. Real world problems are often analyzed with assuming unchangeable input parameters. However, in practice, input data is usually different from mathematical model assumptions. Therefore, these assumptions lead to answers that are far from the optimality and even feasibility in the real world. Demand, types of costs, capacity and ... Are things which change in facility location and network design problems during the time. Therefore studying and developing network design and facility location model in uncertainty mode is considered as one of the gaps in research in this area so it will be tried to consider gap.

Optimization under "uncertainty mode" is typically considers in two perspectives: 1- random optimization and 2 – robust optimization. In random optimization, uncertain parameters are controlled by the probability distribution function and model seeks to provide a solution that minimizes the expected cost of objective function. In "robust" optimization possibilities are indefinite and random parameters are estimated by discrete scenarios or a range of distances. In the discrete case, for each parameter, based on past experience and the feasibility studies conducted, several numbers are recommended that each are defined as "scenario", and in continuous case, each uncertain parameter is determined with a specific interval. In "robust" problems, the ultimate goal is to minimize worst-case cost or regret, which will be discussed in detail. In this paper, customer demand and facility location costs and operational lines construction are assumed to be uncertain and scenarios will be considered for them.

So in this paper we will provide robust mode of facility location problem-network design in the case without capacity constraints, then model solving, and computational analyses will be discussed.

One of the first relevant researches in this area goes back to a paper that was presented in 1993 by Daskin et al. They have proposed a mixed integer-programming model for facility location problem without capacity and network design constraint (DASKIN; HURTER; VNBUER, 1993).

Melkote proposed facility location/network design models in his doctoral thesis (MELKOTE, 1996). In this thesis, three main models have been proposed by considering various assumptions. In 2001, Melkote and Daskin have presented an integrated model of facility location and transportation network design without capacity constraint (MELKOTE; DASKIN, 2001). This model can be used widely in regional planning, distribution, energy management and etc. Many examples of this model were solved using "CPLEX 3" software and several sensitivity analyzes were performed for model parameters.

Also in 1999, multi-criteria network location problem by considering the views of decision makers, have been introduced by Hamacher et al (2002). This model has been analyzed in both Pareto and lexicographically. The objective function considered in this paper is of "median" type. Meanwhile, a polynomial algorithm is presented to solve the proposed model.

In 2000, it was proved that the transportation network design for an existing tool has the potential to become a spanning tree (BHADURY; CHANDRASEKHARAN; GEWALI, 2000). Also, it has been proved that this problem is an NP-complete problem. This paper has presented by Bhadury et al. In this paper, the authors suggest that an exact method or a heuristic algorithm be proposed to solve this problem.

Another model in which the facility capacity assumed limited, was developed by Melkote and Daskin (2001). In this paper, the problem of capacity constraint, has modeled and a mixed integer-programming model has been proposed for it. Problems with 40 nodes and 160 link candidates were solved by CPLEX 3 software. Computational results for the model showed that compared with capacity constraint model: 1 – transportation and lines costs, contrary impression may be reduced; 2- network will be denser and therefore, as expected, is more expensive; and 3 – all components of the objective function (building links, to facilitate and transport cost) at higher capacity levels, are more sensitive.

Drezner and Wesolowsky (2003) conducted a research entitled “network design: selection and design of links and facility location” (2003). They have considered traffic flow or facility location as objective functions in problems. Lines for a given cost or are made or not. Each link can be unilateral or bilateral. These assumptions were lead to the formation of 4 problems that were solved heuristically, using algorithms: the simulation refrigeration, no search, reduction algorithm and genetic algorithm. A comparison of the results of these algorithms, demonstrates the superiority of genetic algorithms over other algorithms.

An estimation algorithm in order to solve the combination problem of logistic and location network have presented by Ravi and Sinha (2004). In this paper, an integer programming model has proposed to solve the model, and the gap to the optimum solution, has been studied. As for future research, the authors suggested that the problem be developed for the case costs are not linear or in the case of multi-product. Also, investigating the problem in the case facilities have limited capacity were other suggestions from writers.

In a thesis, conducted by Jorgensen (2004) in technical university of Denmark, a series of supply chain models in network mode have proposed and evaluated.

Moreover, a series of complex design distribution network models, have been studied by Ambrosino and Scutella (2005). The models presented in this paper, are important for considering different parts of the supply chain, such as storage, facility location and material transportation, simultaneously and are evaluated as complex models. In this paper, it is assumed that there is only one plant and lower limits for the inventory are not considered.

An estimating algorithm for solving a network location problem in cases where the cost of service for each service provider is limited, has presented by Mabberg and Vygen (2005). Minimizing the total cost of equipment deployment and material transportation was the purpose of this paper.

A re-formulated and flexible logistic network design problem with regard to location, factory and warehouse capacity, selecting a transfer method, allocation of goods range and material flow, comparing two approaches to solve this problem, developing a LP relaxation and enhancing both current approaches, as been done by Cordeau et al (2006). In this paper, it is assumed that both production and distribution are of single-stage.

A dissertation at the university of Maryland has conducted by Si Chen (2007), in which studying four network models in the fields of telecommunication, transportation and supply chain and developing a heuristic algorithm to solve them have been discussed. Heuristic algorithm presented in this paper, was the combination of “MIP” method and “record to record travel” algorithm. In this dissertation, it is assumed that the distances are asymmetric and links do not have capacity constraint.

Cocking (2008), has defined his thesis entitled “solutions to facility location-network design problems”, where different algorithms for solving facility location-network design problem have proposed. In this thesis, facility location-network design problem, considering budget constraints has been investigated and near-optimal upper and lower bounds for the problem have presented. In this paper, some algorithms to solve the problem were proposed and implemented. Greedy algorithm, a locally optimal algorithm, refrigeration simulated meta-heuristic algorithm, variable neighborhood search algorithm and a heuristic algorithm which considers facility location and network design problems separately, were included the proposed methods. The results of the calculations have indicated the main variable neighborhood search algorithm had best performance compared to other methods, as on average reached solutions close to 0.6% of the optimal.

An estimation algorithm with constant factor has proposed by Chen and Chen (2009) to solve the FLND problem. In this paper, it is assumed that links and equipment have capacity constraints. Proposed algorithm of this paper, has achieved from the integration of primary-dual method, Lagrange relaxation, demand clustering and two-factor estimation. As future researches, the authors have suggested further deepening of the proposed combination in order to obtain better solutions and considering the problem for the case where the equipment could not be opened more than once.

2 METHODS

2.1 Modeling the optimization problem for network facility location in uncertainty mode (robust mode)

In this section of the paper, the model of optimization problem for network facility location in uncertainty mode (robust mode) will be presented. This model is based on deterministic model that its formulation has presented by Melkote (1996).

2.2 Assumptions and Parameters

In this section, the formulation of the model, which is a mixed integer, is presented. To facilitate model presenting, the parameters and assumptions are described first. Accordingly, the indices and the parameters have been described in Table 1 and Table 2, respectively.

Table 1 – Index set

Description	Index	Symbol
Set of network nodes	$i, j \in \{1, 2, \ldots, \lvert N \rvert\}$	N
Set of customers	$k \in \{1, 2, \ldots, \lvert N \rvert\}$	K
Set of candidate links	$(i, j) \in L$	L
Set of scenarios	$s \in \{1, 2, \ldots, \lvert S \rvert\}$	S

Facility location and network design model under uncertainty, has a number of decision variables. For example, it must be decided where to locate the facility and which links should be established.

Table 2 – Problem parameters

Description	Symbol
Customer k demand under scenario s	d_k^s
Robustness number	p
Fixed cost of facility opening in node i under scenario s	f_i^s
Construction cost of the link (i, j) under scenario s	c_{ij}^s
Cost of goods unit transportation on link (i, j) under scenario s	t_{ij}^s
Cost of customer flow on link (i, j) under scenario s	$tr_{ij}^{ks} = t_{ij}^s * d_k^s$
Probability of scenario s	q_S
The budget	$\boldsymbol{B}$
Optimal solution of deterministic model under scenario s data	Z_s^*

Also, the flow rate and the level of customer satisfaction in every node for each scenario must be specified. Therefore, decision variables of this model are stated as follow:

$$Z_i = \begin{cases} 1 & \text{If a facility be established at node } i \\ 0 & \text{Otherwise,} \end{cases}$$

$$X_{ij} = \begin{cases} 1 & \text{If link } (i, j) \text{ be established} \\ 0 & \end{cases}$$

Otherwise,

Y_{ij}^{ks} Customer ‘s demand rate at link (*i,j*) under scenario *s*

W_{i}^{ks} The demand rate that is satisfied by active facility at node *i*, for customer *k* under scenario *s*

This model has some assumptions that they have considered the following (these are the same assumptions that Daskin, Hurter and Vanbuer (1993) have considered):

- Each node has a certain amount of demand.
- The facility will be established only on the nodes.
- At each node, only one facility can be established.
- Network provides a mode of facility-customer.
- Facility capacity is unlimited.
- Nodes are connected by a direct link.
- Conditions of uncertainty affect many parameters, to solve this problem, different scenarios have been used.
- Weight is equal for all scenarios.

2.3 The main objectives of mathematical model are:

- Optimal location of a limited number of facilities.
- Reducing maintenance and transportation costs.
- Reducing construction costs.
- Obtaining a reliable solution for situations where uncertainty is involved.
- Minimizing the expected cost of each scenario.
- Determining the best network for the transport of goods.

2.4 Limitations of this model are as follows:

- Each customer should be fully provided by other facility.
- Input and output flow rate to each node should be equal.
- If a facility be established in one node, this nod can’t be satisfied by other nodes.
- Each node can satisfy others only in case, which the facility is located on it.

- Flow between two nodes can exist if the communication link is established between them.
- The number of facility, which should be established, is limited.
- The problem parameters are under uncertainty conditions.

2.5 Presented mathematical model

According to the previous definitions and considered assumptions, the model will be as follows (Equation 1):

$p-SUFLNDP$

$$Min\ :\ \sum_{s\in S} q_S * \{ \sum_{(i,j)\in L} \sum_{k\in N:k\neq i} tr_{ij}^{ks} * Y_{ij}^{ks} + \sum_{(i,j)\in L} tr_{ij}^{is} * X_{ij} + \sum_{(i,j)\in L} X_{ij} * C_{ij}^{s} + \sum_{i\in N} Z_i * f_i^{s} \} \tag{1}$$

Considering (Equation 2, 3, 4, 5, 6, 7, 8, 8, 10, 11, 12, 14):

$$Z_i + \sum_{j\in N} X_{ij} = 1; \qquad \forall i\in N, (i,j)\in L, s\in S \tag{2}$$

$$X_{ki} + \sum_{j\in N:j\neq k} Y_{ji}^{ks} = \sum_{j\in N:j\neq k} Y_{ij}^{ks} + W_i^{ks}; \quad \forall i,k\in N: i\neq k, (k,i),(i,j)\in L, \forall s\in S \tag{3}$$

$$\sum_{j\in N:j\neq k} Y_{ji}^{ks} = \sum_{j\in N} Y_{ij}^{ks} + W_i^{ks}; \qquad \forall i,k\in N: i\neq k, (i,j)\in L, (k,i)\notin L, \forall s\in S \tag{4}$$

$$Z_k + \sum_{i\in N:i\neq k} W_i^{ks} = 1; \qquad \forall k\in N, s\in S \tag{5}$$

$$Y_{ij}^{ks} \le X_{ij}; \qquad i,j,k\in N, s\in S, i\neq k, \forall (i,j)\in L \tag{6}$$

$$W_i^{ks} \le Z_i; \qquad \forall i,k\in N: i\neq k, s\in S \tag{7}$$

$$X_{ij} + X_{ji} \le 1; \qquad \forall (i,j)\in L \tag{8}$$

$$\sum_{i\in N} Z_i = r; \tag{9}$$

$$\sum_{(i,j)\in L} \sum_{k\in N:k\neq i} tr_{ij}^{ks} * Y_{ij}^{ks} + \sum_{(i,j)\in L} tr_{ij}^{is} * X_{ij} + \sum_{(i,j)\in L} X_{ij} * C^{s}_{ij} + \sum_{i\in N} Z_i * f_i^{S} \le (1+p) * Z_s^{*} \quad \forall s\in S \tag{10}$$

$$Y_{ij}^{ks} \ge 0, \qquad \forall (i,j)\in L, k\in N, s\in S: k\neq i \tag{11}$$

$$X_{ij} \in \{0,1\}, \qquad \forall (i,j)\in L, k\in N, s\in S: k\neq i \tag{12}$$

$$W_i^{ks} \geq 0, \qquad \forall i,k \in N : k \neq i, s \in S \tag{13}$$

$$Z_i \in \{0,1\}, \qquad \forall i,k \in N : k \neq i, s \in S \tag{14}$$

In this model, Equation (1) represents the objective function. This objective function minimizes the expected value of the transportation costs. This function is looking for answers that produce the lowest expected value for transportation costs, regarding to their robustness. The first part of the objective function considers transportation costs for nodes that are not directly related and the second part is for direct transmission between two nodes that have direct links between each other. This has been done to have a stronger formulation. In this model we have x=y, since x is an integer, but y is a positive number, but it should be noted that y will also be an integer in the optimal solution.

Constraint (2) (Equation 2) ensures that the demand for each customer is fully satisfied. This can be done by the facility that is established on the same node or be transferred to other nodes.

Constraints (3 and 4) (Equation 3 and 4) examine the equilibrium condition. In this constraint, the flow rate comes into node i will be equal to the amount of current that comes out of it. In constraint (3) (Equation 3) , the balance for node *i* is examined in case the customer K is directly connected to this node. While in constraint (4) (Equation 4) the connection is created by passing through other nodes.

Constraint (5) (Equation 5) is to each demand be fully satisfied. Also, if a facility establish in a node, not be transferred to another node.

Constraint (6) (Equation 6) implies that the flow from node *i* to j can exist if a link is established between these two links. Similarly, in constraints (7) (Equation 7) node *i* can satisfy demand of customer k, if a facility be established it. Since this study is based on making one-way links, constraint (8) (Equation 8) is written. Constraint (9) (Equation 9) states the number of facility that must be established. This means that a maximum of r facilities can be established.

Constraint (10) (Equation 10) holds the p-robust condition. In this constraint, the costs of transportation, line construction and facility establishment in each scenario under the current solution must be less than $(1+p)*Z_s^*$.

These constraints make obtained results be robust. If budget constraint be applied to design problem, constraint (10) (Equation 10) can be written in the following form:

$$\sum_{(i,j)\in L}\sum_{k\in N:k\neq i} tr_{ij}^{ks} * Y_{ij}^{ks} + \sum_{(i,j)\in L} tr_{ij}^{is} * X_{ij} \leq (1+p) * Z_s^* \qquad \forall s \in S \tag{10}$$

In this case, the objective function (1) (Equation 1) and constraint (9) (Equation 9), will be respectively changed to the following form:

$$Min : \sum_{s\in S}\sum_{(i,j)\in L}\sum_{k\in N:k\neq i} q_S * tr_{ij}^{ks} * Y_{ij}^{ks} + \sum_{s\in S}\sum_{(i,j)\in L} q_S * tr_{ij}^{is} * X_{ij} \tag{1}$$

$$\sum_{(i,j)\in L} X_{ij} * C^s_{ij} + \sum_{i\in N} Z_i * f_i^S \leq B; \ \forall s \in S \tag{9}$$

Constraints (9) (Equation 9) represents the amount of budget that can be spent on facility establishment and lines constructing. In this constraint, the amount of available budget is shown by mark b. Constraints (11-14) (Equation 11-14) represent problem's decision variables, which have been studied previously.

Optimal solution for each scenario is obtained from the formula that presented in following. This means that, it is only necessary, the following deterministic facility location and network design problem be solved optimally, by the data of each scenario.

$$Min : \sum_{(i,j)\in L}\sum_{k\in N:k\neq i} tr_{ij}^{k} * Y_{ij}^{k} + \sum_{(i,j)\in L} tr_{ij}^{i} * X_{ij} + \sum_{(i,j)\in L} X_{ij} * C_{ij} + \sum_{i\in N} Z_i * f_i \tag{1}$$

Considering (Equations):

$$Z_i + \sum_{j\in N} X_{ij} = 1; \qquad \forall i \in N, (i,j) \in L \tag{2}$$

$$X_{ki} + \sum_{j\in N:j\neq k} Y_{ji}^k = \sum_{j\in N:j\neq k} Y_{ij}^k + W_i^k; \qquad \forall i,k \in N : i \neq k, (k,i), (i,j) \in L \tag{3}$$

$$\sum_{j\in N:j\neq k} Y_{ji}^k = \sum_{j\in N} Y_{ij}^k + W_i^k; \qquad \forall i,k \in N : i \neq k, (i,j) \in L, (k,i) \notin L \tag{4}$$

$$Z_k + \sum_{i\in N:i\neq k} W_i^k = 1; \qquad \forall k \in N \tag{5}$$

$$Y_{ij}^{k} \leq X_{ij}; \qquad i,j,k \in N, i \neq k, \forall (i,j) \in L \tag{6}$$

$$W_{i}^{k} \leq Z_{i}; \qquad \forall i,k \in N : i \neq k \tag{7}$$

$$X_{ij} + X_{ji} \leq 1; \qquad \forall (i,j) \in L \tag{8}$$

$$\sum_{i \in N} Z_{i} = r; \tag{9}$$

$$Y_{ij}^{k} \geq 0, \qquad \forall (i,j) \in L, k \in N : k \neq i \tag{10}$$

$$X_{ij} \in \{0,1\}, \qquad \forall (i,j) \in L, k \in N : k \neq i \tag{11}$$

$$W_{i}^{k} \geq 0, \qquad \forall i,k \in N : k \neq i \tag{12}$$

$$Z_{i} \in \{0,1\}, \qquad \forall i,k \in N : k \neq i \tag{13}$$

2.6 Upper bound of the problem:

In many of methods to solve problem, including branch and bound based solution methods and also approximate methods, having an upper bound to the problem, is highly required. For this problem, a safe upper bound can be obtained based on what (DASKIN, 1993) has introduced. To obtain this upper bound, the Equation 15 provided below is used.

$$\text{the upper bound} = \sum_{\forall s \epsilon S} \mathrm{q_s}(1 + \mathrm{p})\mathrm{z}_{\mathrm{s}}^{*} \tag{15}$$

In the next section, the presented model has been tested on 10 different randomly generated sample problems and these problems have been fully described and, further computational results and sensitivity analysis on the parameters have been presented.

3 RESULTS AND ANALYSIS

3.1 Solving the model, the computational results and sensitivity analysis

In this paper, to investigate the performance of the proposed model, 10 different sample problems are solved in multiple dimensions. These problems have been solved by software GAMS and CPLEX 10.2 solver. CPLEX algorithm uses branch and bound and cutting plane methods in solving different integer optimization problems. Using cutting plane in solving models requires much less running time and this is clearly seen in the sample problems.

3.2 Generating data and sample problems

For all problems, sample links are new and we already do not have an established link. These variations, as shown in the Table 3.

Table 3 – Variety of problems

Sample problem	Number of nodes	Number of links	Number of scenarios	Number of facilities
TP1	5	18	5	1
TP2	10	44	5	2
TP3	10	38	5	2
TP4	20	108	5	2
TP5	20	122	5	3
TP6	40	264	5	3
TP7	40	274	5	6
TP8	40	324	5	6
TP9	60	410	5	9
TP10	60	360	5	9

Location of network nodes has selected randomly in a 100*100 environment. For each problem, nodes demand under scenario 1, has been selected randomly from interval [0.10000]. For the next scenarios, scenario 1 data have been multiplied by a random number between 0.5 and 1.5.

This means that the first scenario estimations for the demand may be increased or decreased by 50%. These values have been rounded to the nearest integer. Line construction costs for the first scenario, is considered the Euclidean distance between nodes multiplied by the unit cost of link construction ($C_{ij}^{s} = u * t_{ij}^{s}$). For these parameters, also the data in the first scenario has multiplied by a random number between [0. 5, 1.5], to additional scenarios be obtained. Since the unit cost of the link construction parameter doesn't has a significant impact on solving time it has adjusted to establish an equilibrium between line construction costs, facility location costs and transportation costs. Hence, in this study, the unit cost of line construction has been considered equal to 7.35 (what Melkote has considered). In this study, probability or weight of scenarios are considered equal. This means that all scenarios occur with equal probabilities, which are calculated by the following Equation 16:

$$q_s = \frac{1}{|S|}; \forall s \in S \tag{16}$$

But an alternative formulation can be as follows, which the weight is calculated for each scenario based on its demand. But overall, the expected value of probabilities will be as the previous Equation 17.

$$q_s = \frac{\sum_{k \in N} d_k^s}{\sum_{s \in S} \sum_{k \in N} d_k^s} \quad \forall s \in S \tag{17}$$

According to the second presented model, the optimal solution is obtained from the individual scenarios firstly. These have marked as s1 to s5 in Table 3. In the column opposite each sample problem, required time and also obtained optimal solution have been reported, titled "time" and "solution". In this table, the column entitled "expected value of scenarios" represents the weighted average of solutions obtained from solving each scenario. This means that the solution obtained for each scenario (both in terms of time and the objective function solution), has multiplied by the probability of that scenario and is reported in this column.

As it was said, and in general for p-robust problems, the result of solving each scenario has provided as an input parameter for the model. By placing, this results in the model and solving it by software GAMS the results have been reported in column "P-SUFLNDP". The results indicate an increase in the solution time and imposed costs. To better illustrate the increase in solution time and cost compared with expected value of scenarios, in the last column of the table change percentages or the increase percentages have been reported for these two important. To calculate the change percentages the following Equation 18 has been used.

$$Chang\ percentages = (obtained\ results\ from\ model - expected\ value\ of\ scenarios)/obtained\ results\ from\ model \tag{18}$$

As the robustness number has considered equal to 0.6, the results of calculating upper bound for the problem is presented in the Table 4.

Table 4 – Change percentages for 10 sample problems

TP		S1	S2	S3	S4	S5	Expected value of scenarios	P-SUFLNDP	Change percentages
TP1	Time	3.23	3.12	3.03	3.01	3.12	3.01	3.46	**13.01**
	Solution	715893	628806	760846	785783	551183	687552	717141	**3.96**
TP2	Time	3.55	3.25	3.24	3.14	3.23	3.28	3.59	**8.64**
	Solution	419762	377507	344329	410338	304112	371209	419591	**11.53**
TP3	Time	3.42	3.24	3.14	3.15	3.09	3.21	3.64	**11.81**
	Solution	642786	502803	705735	534884	578184	592787	649876	**8.77**
TP4	Time	3.44	3.43	4.43	3.34	3.31	3.59	4.39	**18.22**
	Solution	786412	9	649447	625325	722123	714287	759579	**5.96**
TP5	Time	3.55	3.45	3.42	3.44	3.36	3.44	4.67	**26.34**
	Solution	1024778	892873	930631	964983	980824	958817	1075862	**10.88**
TP6	Time	5.57	5.13	4.89	5.29	4.96	5.19	15.31	**66.10**
	Solution	1236173	1008823	994832	1147878	1137306	1105002	1194176	**7.47**
TP7	Time	5.43	5.51	5.07	5.15	5.14	5.26	18.76	**71.96**
	Solution	967147	859013	762131	801393	829521	843841	936880	**9.93**
TP8	Time	5.87	5.14	5.28	5.07	5.07	5.29	19.02	**72.19**
	Solution	861230	751291	785925	767212	735558	780243	855075	**8.75**

TP		S1	S2	S3	S4	S5	Expected value of scenarios	P-SUFLND P	Change percentages
TP9	Time	9.67	9.05	9.11	9.02	8.80	9.13	38.31	**76.17**
	Solution	1211188	1048439	1012483	1096800	1025183	1078819	1177734	**8.40**
TP10	Time	15.45	8.84	8.67	10.67	8.74	10.47	68.58	**84.73**
	Solution	1577692	1310192	1364415	1438615	1305195	1399222	1545093	**9.44**

With respect to change percentages, it can be said that the time has increased as much as 44.92 percent on average but for the costs, it shows an increase of 7.74 percent. Also, we have depicted the results for the change percentages in the Figure 1. As is clear from the Figure 1, as the problem dimension increases, the time grows more rapidly. As for sample, problem 1 increase percentage is about 13 percent but for sample problem, 1 is roughly 84 percent. However, the objective function has been little change. It shows with increasing the problem dimension, how complexity of solving the problem rises and need more time to solve.

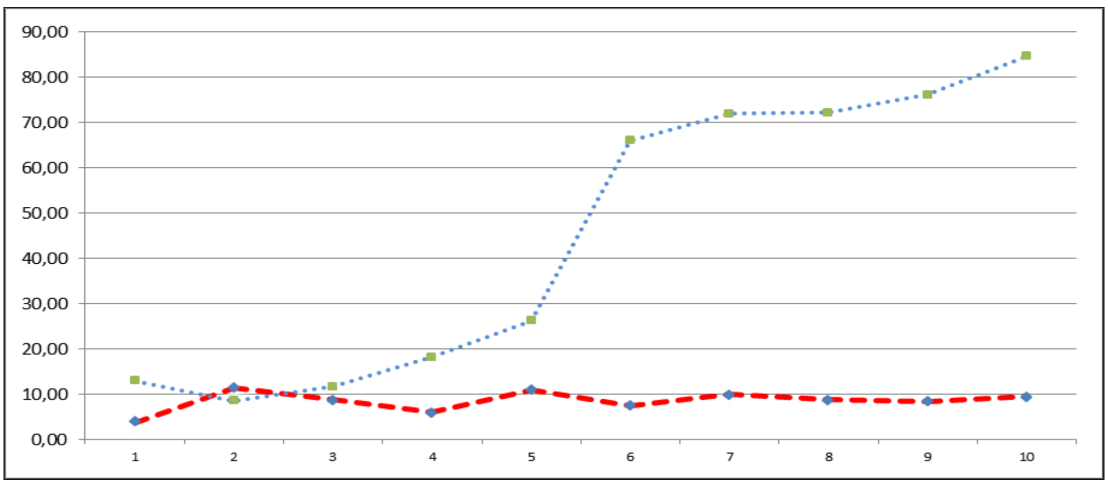

Figure 1 – Change percentages of objective function (red colored curve) and increase percentage of the time (blue colored curve) as problem dimension increases

3.4 Calculating upper bound of problem according to the proposed formula

As the robustness number has considered equal to 0.6, the results of calculating upper bound for the problem is presented in the Table 5. In the fourth column, the answers are calculated by the following Equation (19).

$$Change\ percentages\ =\ (upper\ bound\ -\ results\ from\ software\ solving)/upper\ bound \qquad (19)$$

Table 5 – The results of calculating upper bound for 10 sample problems

Sample problem	Upper bound	Software results	Distance of solutions
TP1	1101604	717141	34.90
TP2	593935	419591	29.35

TP3	948605	649876	31.49
TP4	1142860	759579	33.54
TP5	1534108	1075862	29.87
TP6	1768004	1194176	32.46
TP7	1350146	936880	30.61
TP8	1248389	855075	31.51
TP9	1726110	1177734	31.77
TP10	2238755	1545093	30.98
Average			**31.65**

On average upper bounds are about 31.65% more than optimal solutions of the problems. But it is noteworthy that as the robustness number decreases to its minimum value, this distance will decrease.

3.5 Sensitivity analysis

The presented model has many parameters so it is necessary that the sensitivity of these parameters to be analyzed. To do this, the sample problem tp5 has been considered. In this section, the results of the sensitivity analysis have presented.

3.6 Sensitivity analysis of number of scenarios

In this section, we have tried to examine complexity and variation of the objective function as the number of considered scenarios increases. For this, we have increased the number of scenarios ranging from 2 to 25. Obtained results are reported in the Table 6. It is evident from the table, as the number of scenarios increase, solution time and objective function value increase too.

Table 6 – Sensitivity analysis and objective function variations as the number of scenarios increase

Number of scenarios	Time	Solution
2	3.65	388194
3	4.1	606812
4	4.23	837475
5	4.67	1075862
6	5.19	1254661
7	5.49	1434997
8	5.82	1669733
9	6.19	1902740
10	6.69	2106419
11	7.02	2325461
12	7.68	2521142
13	7.79	2748947
14	8.48	2978461
15	8.05	3143504
16	14.26	3372402
17	9.2	3573002

Number of scenarios	Time	Solution
18	9.54	3745534
19	10.15	3993467
20	11.16	4217394
21	10.61	4425823
22	12.42	4681204
23	20.66	4907975
24	23.79	5090648
25	24.32	5285980
2	3.65	388194

For obtained times and resulted solutions, by increasing the number of scenarios, the two charts (Figure 2 and 3).

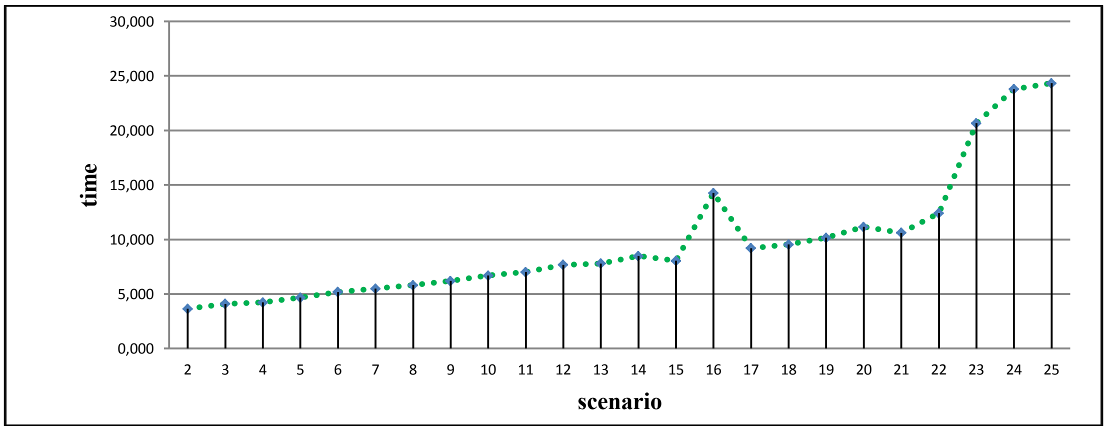

Figure 2 – Increasing amount of time for increasing number of scenarios

These figures clearly show that how complexity of the problem increases by increasing the number of scenarios.

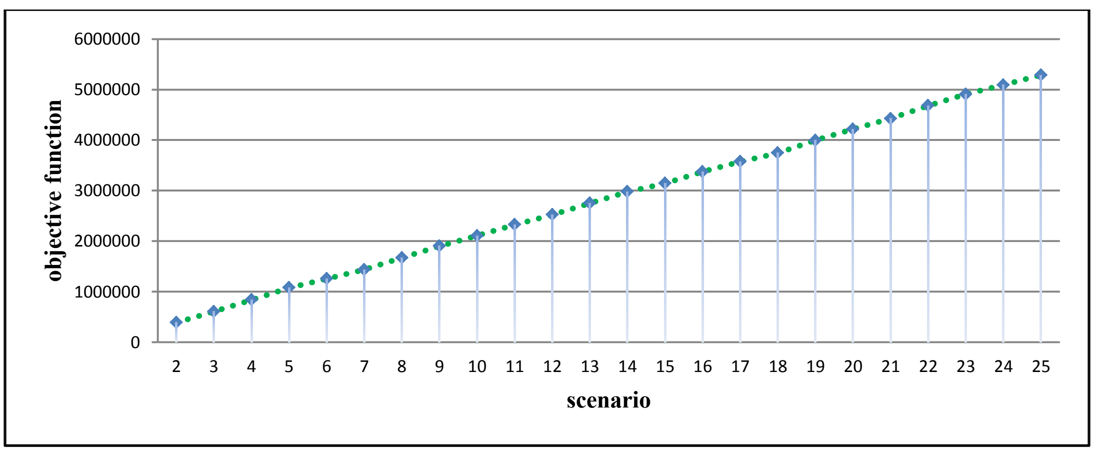

Figure 3 – Increasing the objective function values for increasing number of scenarios

3.8 Sensitivity analysis of number of facilities

For problem tp5, solution time changes and objective function value or the costs has been studied for changes in the number of facilities. The results have presented in the Table 7.

Table 7 – Sensitivity analysis of the time and objective function for changes in the number of facilities

Number of facilities	Time	Solution
1	6.01	INF
2	5.35	INF
3	4.67	1075862
4	4.81	784346
5	5.2	658402
6	5.33	548141
7	5.36	439509
8	4.41	344319
9	5.49	265458
10	4.53	209199
11	4.59	182004
12	4.53	167502
13	4.44	153599
14	4.53	140678
15	4.44	129260
16	4.52	118747
17	4.59	113222
18	4.66	109748
19	4.51	111941
20	4.54	118137

With the increase in the number of facilities that must be established, transportation costs and the cost lines constructing costs will be reduced but the costs associated with the facility establishing will increase. In general, it can be concluded that with the increasing number of facilities, the expected value of costs and solving time will reduce.

3.9 Sensitivity analysis of the robustness number

One aim of such modeling is large reduction in robustness number led in only a small change in the objective function. In this model, the same thing is also true. To show this theorem, we have considered the sample problem tp5. First, we have solved the model for infinity numeric value of robustness number then each scenario have been solved for obtained result. Finally, this obtained result has been saved. Number 0.00001 has subtracted from the maximum allowable amount of robustness number and the problem has solved again. This process continues as long as the problem be unjustified for robustness number. The results are reported in the Table 8.

Table 8 – Sensitivity analysis of objective function for robustness number changes

Robustness number	**0.2372**	**0.2367**	**0.2283**	**0.2187**	**0.2106**
Objective function	1075862	1076269	1078148	1081282	1085003

As shown in the Table 8 the first change in the expected value of cost happens to robustness number of 0.2372. And until reaching its minimum value of 0.2106 in which the model is justified, in the other three steps objective function increase.

As can see in Figure 4, which have been obtained from table 8, by reducing robustness number as much as 11.21% , only a 0.85% increase in the value of objective function has established.

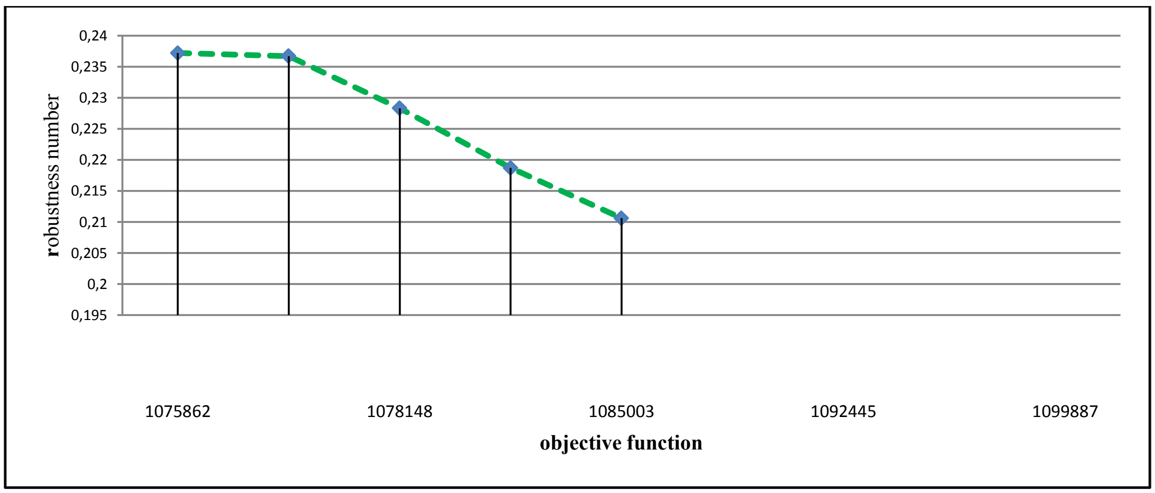

Figure 4 – Objective function changes for robustness number changes

It is noteworthy that with the reduction in the robustness number, problem complexity, solution time and the expected value of the costs increase.

4 Conclusions

In this paper facility location and network design, problem optimization model under conditions of uncertainty (robust mode) was presented. This model minimizes expected costs of facility establishment and the construction of communication lines in a manner that the obtained solution to be robust. Basically, in robust optimization the worst possible case will optimize. Moreover, in stochastic optimization the expected cost minimizes.

However, this paper presented a model for the under study problem which satisfy both goals through minimizing the expected costs with robustness constraint. This means that the provided solution by the model is stable and minimizes the expected costs. Therefore, the model was tested on 10 randomly generated samples and several computational analysis were done and the effect of model parameters on model complexity and behavior measured and evaluated.

Given the research gaps in this area, developing fuzzy and stochastic models for network location problems considering the link cost in accordance with the flow volume can be mentioned as topics for future research.

DESENVOLVIMENTO DO MODELO DE LOCALIZAÇÃO DE REDE NO MODO DE INCERTEZA (MODO ROBUSTO)

RESUMO: Nesta pesquisa, é discutido o problema de localização das instalações – incertezas no projeto de rede em modo robusto. Nesse contexto um modelo será desenvolvido, para que a incerteza dos parâmetros, tais como, a demanda e o problema dos vários custos são considerados. O projeto de localização e instalação de rede, ao contrário dos modelos de localização de instalação clássica, os quais assumem que a estrutura de rede é pré-definida e previamente especificada, também deve decidir sobre a estrutura da rede. Isto tem sido utilizado em muitas aplicações reais, tais como a rede de estradas, sistemas de comunicação, etc. e principalmente na localização e instalações da rede principal, considerou-se importante a necessidade de modelos de projeto simultâneos e de otimização simultâneas para atender os itens mencionados. Diferentes abordagens têm sido desenvolvidos na literatura de otimização com incerteza. Entre elas, otimizações robustas e estocásticas são as mais conhecidas. Para lidar com as incertezas e a modelagem do problema, foi utilizada a abordagem de investigação robusta de otimização. Além disso, foram usadas amostras aleatórias geradas, o modelo proposto foi testado e a análise computacional foi apresentada para vários parâmetros.

Palavras-chave: Localização das instalações. Projeto de rede. Otimização robusta (sólida). *Minimizing the Maximum Regret*.

REFERENCES

AMBROSINO, D.; SCUTELLA M.G. Distribution network design: new problems and related models. **European journal of operational research**, v. 165, p. 610-624, 2005.

BHADURY, J.; CHANDRASEKHARAN, R.; GEWALI, L. Computational complexity of integrated models of network design and facility location. **Southwest Journal of Pure and Applied Mathematics**, n. 1, p. 30-43, 2000.

CHEN, C. **A study of four network problems in transportation, telecommunications and supply chain management**. University of Maryland, USA, 2007.

CHEN, X.; CHEN, B. Approximation algorithms for soft-capacitated facility location in capacitated network design. **Algorithmica**, v. 53, p. 263-297, 2009.

COCKING C. **Solutions to facility location–network design problems**. Thesis (Doctor of philosophy), University of Heidelberg, 2008.

CORDEAU, J.F.F.; PASIN, M.M.; SOLOMON. An integrated model for logistics network design. **Proceeding…,** Annals of operations research, v. 144, n. 1, p. 59-82, 2006.

DASKIN, M.S; HURTER A.P.; VANBUER, M.G. **Toward an integrated model of facility location and paper**. Transportation Center, Northwestern Transportation Network design. Working university, 1993.

DREZNER Z.; WESOLOWSKY, G.O. Network design: selection and design of links and facility location: part a. **Transportation research**, v. 37, p. 241-56, 2003.

HAMACHER, H.W.; LABBE, M.; NICKEL, S.; SKRIVER, A. Multicriteria semi-obnoxious network location problems (msnlp) with sum and center objectives. **Proceeding…,** Annals of operations research, v. 110, p. 33-53, 2002.

JORGENSEN, H.J. **Supply chain models and their applicability**. Thesis, Technical University of Denmark, Denmark, 2004.

MABBERG, J.; VYGEN, J. Approximation algorithms for network design and facility location with service capacities, 2005.

MELKOTE, S. **Integrated models of facility location and network design**. Doctor of Philosophy, Evaston, Illinios: Northwestern University, 1996.

MELKOTE, S.; DASKIN, M.S. An integrated model of facility location and transportation network design: part a. **Transportation research**, v. 35, n. 5, p. 15-38, 2001.

MELKOTE, S.; DASKIN, M.S. Capacitated facility location-network design problems. **European journal of operational research**, v. 129, n. 3, p. 481-95, 2001.

RAVI, R.; SINHA, A. **Approximation algorithms for problems combining facility location and network**, 2004.

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